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A Phase Model of Earthquake Motions based on Stochastic Differential Equation

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Abstract

In this paper, a method is proposed to simulate Group Delay Time (GDT) of earthquake ground motion by using Stochastic Differential Equation (SDE). The random characteristic of GDT is expressed by a stochastic differential equation whose mean and variance processes are defined by ordinary differential equations. An algorithm is developed to identify the coefficients of the ordinary differential equations. Regression surfaces of the coefficients are developed as functions of earthquake magnitude, epicentral distance and frequency. The Milstein approximation scheme is used to solve the stochastic differential equation of GDT. The efficiency of the developed method is demonstrated by comparing the simulated result with the original one.

Keywords: *phase spectrim, group delay time, stochastic differential equation, simulation, modeling*

1. Introduction

It is known that phase spectrum is an important characteristic of earthquake motion. Although amplitude and phase spectrum are essential to simulate an earthquake motion, there have been fewer research efforts conducted on the modeling of phase spectrum comparing with the modeling of amplitude spectrum.

Several pioneer studies explored the stochastic characteristics of earthquake motion through analysis of phase characteristics. Ohsaki (1979) discussed the probability distributions of Fourier phase angles and phase differences of earthquake motions. Nigam (1982) derived an analytic expression for the distribution of phase differences of a random process corresponding to Gaussian white noise multiplied by a time-domain shaping function and remarked that the distribution is not normal. Katsukura and Izumi (1983) showed that average arrival time and duration of earthquake motion can be evaluated by the mean and standard deviation of Group Delay Time (GDT). Jin and Liao (1993) pointed out the differences between the phase spectrum and the main value of phase spectrum ranging from 0 to -2π. Kimura (1986) presented a method to simulate earthquake motion by controlling the group delay time. Shrikhande *et al.* (2001) cast the problem of characterizing phase spectra in the form of a constrained nonlinear programming problem and modeled the phase curve of a earthquake ground motion by a piecewise-linear generic curve superimposed with zero-mean Gaussian residual phases to capture the characteristics of the time-domain envelope of the earthquake ground motion. Boore (2003) listed several advantages to using group delay rather than phase differences. Since the average value of the group delay time within a certain frequency band with the central frequency ω expresses the arrival time of a wave component with frequency ω and the distribution width of the group delay time is related to the duration of the time history of the wave component, its modeling is much easier than direct modeling of phase spectrum.

The parameters that control group delay time of earthquake motion are modeled as function of circular frequency, epicentral distance and earthquake magnitude. Several studies developed the attenuation relationship related to group delay time of earthquake motion. Ishii (1987) put forward attenuation relationships of the mean group delay time and its standard deviation for entire time history of earthquake motion. Satoh (1996) derived attenuation relationships of the mean group delay time and its standard deviation for fairly long period earthquake motion by using a wave form processed through a narrow band passed filter with the frequency of 0.02Hz. Sato and Nishimura (2002) studied the phase characteristic of earthquake motions and developed a method to simulate phase spectrum according to the compact support of Meyer wavelet. In their studies regression relation that expresses the mean and standard deviation of the group delay time as functions of epicentral distance and earthquake magnitude was proposed.

This paper presents a new method to simulate group delay time of earthquake motion by using Stochastic Differential Equation

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(SDE). The stochastic differential equation whose mean and square processes are defined by ordinary differential equations is assumed to express the stochastic characteristic of group delay time. First, a method to calculate group delay time of earthquake motion is proposed. Using dataset of observed earthquake motions, a database of group delay time is developed to be used for regression analysis of coefficients controlling stochastic differential equations. Second, an algorithm is developed to identify the coefficient functions that controlling the stochastic differential equation. After storing these coefficients into parameter dataset, regression surfaces of these coefficients are established as functions of circular frequency, epicentral distance and earthquake magnitude. In the end, the Milstein approximation scheme is used to solve the stochastic differential equation. The efficiency of the simulation model of group delay time is demonstrated by comparing the simulation result with the original one.

2. GDT Model based on Stochastic Differential Equation

In this section, a new method to calculate GDT of earthquake motion is proposed. And then a GDT model is established based on stochastic differential equation.

2.1 Calculation of GDT of Observed Earthquake Motion using Fourier Transform

There are several methods to calculate GDT of earthquake motions. In this section, a method using Fourier transform to calculate GDT is proposed and a dataset of GDT of observed earthquake motions is developed.

When a time history of earthquake motion $z(t)$ is given, its Fourier transform $Z(\omega)$ is expressed by:

$$Z(\omega) = \int_{-\infty}^{\infty} z(t)e^{-i\omega t}\mathrm{d}t = R(\omega)+iI(\omega) = A(\omega)e^{i\varphi(\omega)} \tag{1}$$

in which, ω is the circular frequency, $R(\omega)$ and $I(\omega)$ are real and imaginary parts of $Z(\omega)$, $i^2 = -1$, $A(\omega)$ and $\varphi(\omega)$ are amplitude and phase spectrum of $z(t)$. The group delay time $\xi\omega$ at the circular frequency ω is defined by:

$$\xi_\omega = \frac{d\varphi(\omega)}{d\omega} \tag{2a}$$

The calculation of group delay time is usually based on the following formula:

$$\xi_\omega = \frac{d\varphi(\omega)}{d\omega} = \frac{\dfrac{dI(\omega)}{d\omega}R(\omega) - \dfrac{dR(\omega)}{d\omega}I(\omega)}{R^2(\omega)+I^2(\omega)} \tag{2b}$$

To calculate the first derivative of function $F(\omega)$ with respect to circular frequency, the Fourier and its inverse transform relations are used:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}d\omega \tag{3a}$$

$$\frac{dF\omega}{d\omega} = \int_{-\infty}^{\infty} (-it)f(t)e^{-i\omega t}dt \tag{3b}$$

For calculation using Fourier and its inverse transforms, Fast Fourier Transform (FFT) algorithm is usually used. In this paper, a record of the West Off Fukuoka Prefecture Earthquake from Japanese databank of KiK-net (FKOH03-EW) is selected as the example. Fig. 1(a) shows its time history. Using the method given above, its GDT is obtained (see Fig. 1(b)). Considering the fact that the result of FFT is symmetrical, only half of the length of the obtained data is retained.

Using dataset of observed earthquake motions from Japanese databank of KiK-net, a database of group delay time is developed to be used for regression analysis of parameters controlling the stochastic differential equation defined in the following section. Those parameters are assumed as functions of earthquake magnitude, epicentral distance and frequency.

2.2 Stochastic Differential Equation to Characterize a GDT

The stochastic characteristic of group delay time of earthquake motion is assumed to be expressed by the following stochastic differential equation:

$$d\xi_\omega = \{c_1(\omega)\xi_\omega + c_2(\omega)\}d\omega + \{\sigma_1(\omega)\xi_\omega + \sigma_2(\omega)\}dB_\omega \tag{4}$$

where, $c_1(\omega)$, $c_2(\omega)$, $\sigma_1(\omega)$ and $\sigma_2(\omega)$ are coefficient functions to be identified. B_ω is Wiener process. $\{c_1(\omega)\xi_\omega + c_2(\omega)\}$ is usually called drift term which brings the process variable being modeled back to some equilibrium level (mean values) when $c_1(\omega) < 0$. $\{\sigma_1(\omega)\xi_\omega + \sigma_2(\omega)\}$ is diffusion coefficients. Every time the diffu-

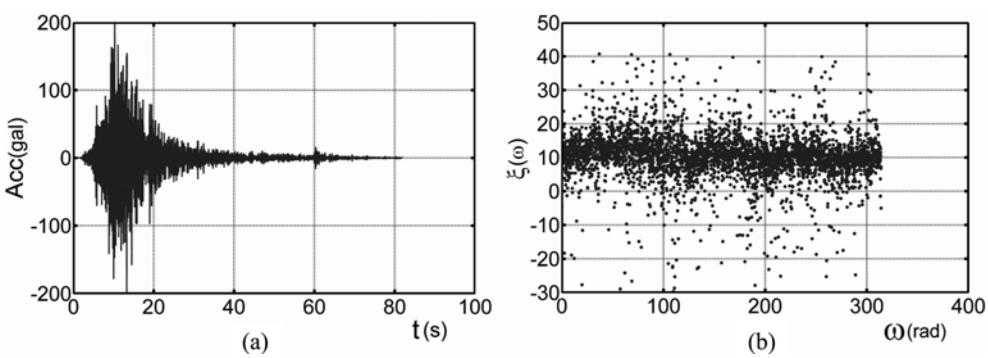

Fig. 1. (a) Time History of FKOH03-EW, (b) Group Delay Time of the Earthquake Motion

sion coefficients gives the process variable a push away from the equilibrium level, the drift term will act in such a way that the process variable will start heading back to the equilibrium level.

According to stochastic differential equation theory, the mean of group delay time $\mu_\xi(\omega) = E[\xi_\omega]$ and mean square value of group delay time $q_\xi(\omega) = E[\xi_\omega^2]$ are expressed by the following ordinary differential equations (Mikosch, 1998):

$$\frac{d\mu_\xi(\omega)}{d\omega} = c_1(\omega)\mu_\xi(\omega) + c_2(\omega) \tag{5}$$

and

$$\frac{dq_\xi(\omega)}{d\omega} = \{2c_1(\omega) + \sigma_1^2(\omega)\}q_\xi(\omega) + 2\{c_2(\omega) + \sigma_1(\omega)\sigma_2(\omega)\}\mu_\xi(\omega) + \sigma_2^2(\omega) \tag{6}$$

To determine the coefficient functions appeared in Eq. (4), the data of $\mu_\xi(\omega)$, $\mu_\xi'(\omega) = d\mu_\xi(\omega)/d\omega$, $q_\xi(\omega)$ and $q_\xi'(\omega) = dq_\xi(\omega)/d\omega$ are needed. Here group delay time is assumed to be ergodic weak stationary process in frequency domain. From this assumption the data needed can be calculated by:

$$\mu_\xi(\omega) \cong \bar{\mu}_\xi(\omega) = \frac{1}{2\omega_a}\int_{\omega-\omega_a}^{\omega+\omega_a} \xi(y)dy \tag{7}$$

$$\mu'_\xi(\omega) = \int_{-\infty}^{\infty} (-iy)M_\xi(y)e^{-i\omega y}dy \tag{8}$$

$$q_\xi(\omega) \cong \bar{q}_\xi(\omega) = \frac{1}{2\omega_a}\int_{\omega-\omega_a}^{\omega+\omega_a} \xi^2(y)dy \tag{9}$$

$$q'_\xi(\omega) = \int_{-\infty}^{\infty} (-iy)Q_\xi(y)e^{-i\omega y}dy \tag{10}$$

in which $2\omega_a$ is the width of window to calculate mean values, $M_\xi(y)$ and $Q_\xi(y)$ are inverse Fourier transforms of $\mu_\xi(\omega)$ and $q_\xi(\omega)$. For numerical calculation of $\mu_\xi(\omega)$ and $q_\xi(\omega)$ in the following section, 11 integral points (ω_{i-5}, …, ω_{i-1}, ω_i, ω_{i+1},…, ω_{i+5}, i.e., $\omega_a = 5\Delta\omega$, $\Delta\omega = \omega_i - \omega_{i-1}$) are selected for each frequency of ω_i.

To determine the coefficient functions, several identification algorithms such as the least square method, Kalman filter method and Monte Carlo filter method can be used. For simplicity, the algorithm of the least square method is selected here. The cost functions to be minimized are given from Eqs. (5) and (6) as follows:

$$Q_\mu = \int_0^{\omega_D}\{\mu'_\xi(\omega) - c_1(\omega)\mu_\xi(\omega) - c_2(\omega)\}^2 d\omega \tag{11}$$

$$Q_q = \int_0^{\omega_D}\{q'_\xi(\omega) - \{2c_1(\omega) + \sigma_1^2(\omega)\}q_\xi(\omega) - 2\{c_2(\omega) + \sigma_1(\omega)\sigma_2(\omega)\}\mu_\xi(\omega) + \sigma_2^2(\omega)\}^2 d\omega \tag{12}$$

in which, ω_D is the end circular frequency of concern. Minimization of the cost functions Q_μ and Q_q with respect to coefficient functions $c_1(\omega)$, $c_2(\omega)$, $\sigma_1(\omega)$ and $\sigma_2(\omega)$ can be realized by parameterizing the coefficient functions as follows:

$$c_1(\omega) = f_1(a_1, a_2, \cdots, a_{m_1}, \omega) \tag{13}$$

$$c_2(\omega) = f_2(b_1, b_2, \cdots, b_{m_2}, \omega) \tag{14}$$

$$\sigma_1(\omega) = g_1(\alpha_1, \alpha_2, \cdots, \alpha_{m_3}, \omega) \tag{15}$$

$$\sigma_2(\omega) = g_2(\beta_1, \beta_2, \cdots, \beta_{m_4}, \omega) \tag{16}$$

in which, a_l ($l = 1,\ldots, m_1$), b_l ($l = 1,\ldots, m_2$), α_l ($l = 1,\ldots, m_3$) and β_l ($l = 1,\ldots, m_4$) are parameters controlling the coefficient functions. Substituting Eqs. (13)-(16) into Eqs. (11)-(12), the cost func- tions can be rewritten as

$$Q_\mu = \int_0^{\omega_D}\{\mu'_\xi(\omega) - h_1(a_1, a_2, \cdots, a_{m_1}, b_1, b_2, \cdots, b_{m_2}, \omega)\}^2 d\omega \tag{17}$$

$$Q_q = \int_0^{\omega_D}\{q'_\xi(\omega) - h_2(a_1, a_2, \ldots, a_{m_3}, \beta_1, \beta_2, \cdots, \beta_{m_4}, \omega)\}^2 d\omega \tag{18}$$

Where, h_1 and h_2 are functions of parameters a_l ($l = 1,\ldots, m_1$), b_l ($l = 1,\ldots, m_2$), α_l ($l = 1,\ldots, m_3$) and β_l ($l = 1,\ldots, m_4$).

Based on these equations, the coefficient functions as $c_1(\omega)$, $c_2(\omega)$, $\sigma_1(\omega)$ and $\sigma_2(\omega)$ can be identified from dataset of group delay time.

3. Numerical Simulation of GDT Using SDE

Using the method given in the previous section, the coefficients can be identified with the developed GDT database. In this section a numerical example to identify the coefficients of the SDE is given. And then regression analysis of the coefficients is presented.

3.1 Identification and Regression Analysis of the Coefficients of the SDE

For simplcity, power series are assumed for each coefficient functions:

$$c_1(\omega) = \sum_{l=1}^{m_1} a_1\omega^{l-1}, \quad c_2(\omega) = \sum_{l=1}^{m_2} b_1\omega^{l-1} \tag{19}$$

$$\sigma_1(\omega) = \sum_{l=1}^{m_3} \alpha_1\omega^{l-1}, \quad \sigma_2(\omega) = \sum_{l=1}^{m_4} \beta_1\omega^{l-1} \tag{20}$$

Based on comparison of trial calculation results, $m_1 = m_2 = 3$, $m_3 = m_4 = 6$ are chosen in numerical simulation. And then the parameters a_l ($l = 1,\ldots, m_1$), b_l ($l = 1,\ldots, m_2$), α_l ($l = 1,\ldots, m_3$) and β_l ($l = 1,\ldots, m_4$) can be identified. Also the coefficient functions of $c_1(\omega)$, $c_2(\omega)$, $\sigma_1(\omega)$ and $\sigma_2(\omega)$ can be identified.

To develop regression functions of these four coefficients with relation to epicentral distance (D) and earthquake magnitude (M), observed dataset of the same earthquake motion is needed to ensure homogeneous distribution. Here earthquake motions with 130 and 132 records from different stations for the earthquake of EQ03300-EW with magnitude $M = 7.3$ and the earthquake of EQ00500-EW with magnitude $M = 7.1$ from Japanese databank of K-NET are selected. After storing these calculated coefficient values into dataset, regression surfaces for these coefficients as functions of epicentral distance (D) and circular frequency are developed (Fig. 2). Some values of the regression results of coefficients against epicentral distance (with circular frequency equals 5 Hz) are shown in Table 1. It can be seen from Fig. 2 that

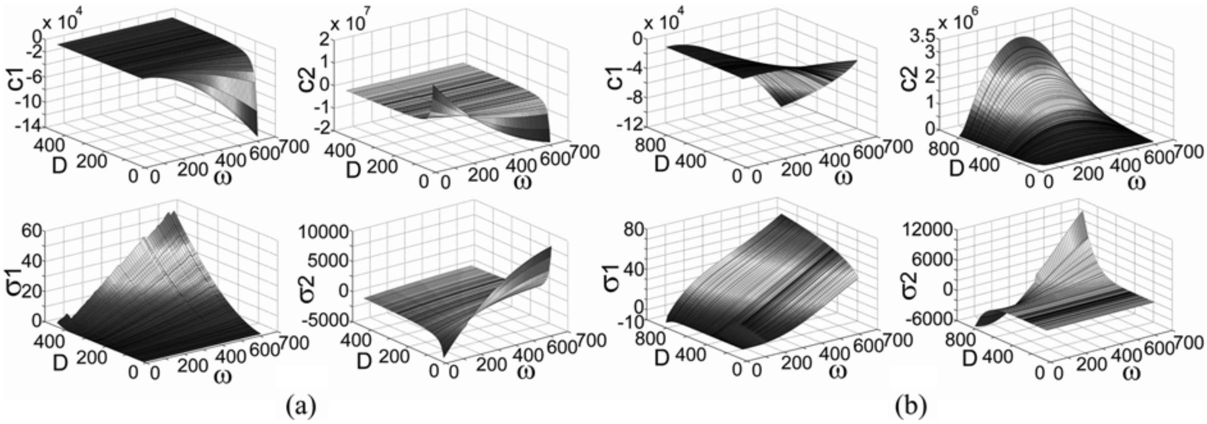

Fig. 2. Regression surfaces for Coefficients as Function of Circular Frequency and Epicentral Distance: (a) M = 7.3, (b) M = 7.1

influence of epicentral distance on coefficients (namely the absolute value of the coefficients) increases as frequency increases.

Similar to the reference (Sato *et al*., 2002), for a certain circular frequency ω_0, the model are assumed as:

$$c_1 = \alpha_1 \cdot 10^{\beta_1 M} D^{\gamma_1}, \quad c_2 = \alpha_2 \cdot 10^{\beta_2 M} D^{\gamma_2} \tag{21}$$

$$\sigma_1 = \alpha_3 \cdot 10^{\beta_3 M} D^{\gamma_3}, \quad \sigma_2 = \alpha_4 \cdot 10^{\beta_4 M} D^{\gamma_4} \tag{22}$$

in which, α_i, β_i, γ_i ($i = 1, 2, 3, 4$) are parameters to be identified. Table 2 shows the regression results of Eq. (21)-(22) for the earthquake of EQ00600-EW with magnitude $M = 7.1$ from Japanese earthquake databank of K-NET.

3.2 Numerical Simulation of the Stochastic Differential Equation of GDT

After the identification of the coefficients, the stochastic differential equation can be solved by using numerical method.

Considering a stochastic differential equation given by:

$$dX_t = a(X_t)dt + b(X_t)dB_t \tag{23}$$

Based on the Milstein approximation (Mikosch, 1998), an integration scheme of this equation is given by:

$$\begin{aligned} X_0^{(n)} &= X_0 \\ X_{t_j}^{(n)} &= X_{t_{j-1}}^{(n)} + a(X_{t_{j-1}}^{(n)})\Delta_j + b(X_{t_{j-1}}^{(n)})\Delta_j B \\ &\quad + \frac{1}{2} b(X_{t_{j-1}}^{(n)}) b'(X_{t_{j-1}}^{(n)}) \{(\Delta_j B)^2 - \Delta_j\} \\ &\quad (j = 1, 2, \cdots, n) \end{aligned} \tag{24}$$

Table 1. Values of Regression Analysis of the Coefficients ($\omega = 5$ Hz)

	D = 25 km	D = 50 km	D = 100 km
c1	-3.692E-04	-1.344E-04	-4.926E-03
c2	-6.597E-06	-2.499E-06	-9.516E-05
σ1	0.251	0.892	3.164
σ2	4.828E-03	2.647E-03	1.454E-03

This scheme can be applied to integrate Eq. (4). Comparing Eq. (4) with Eq. (23) the following equalities are obtained:

$$a(\xi_\omega) = c_1(\omega)\xi_\omega + c_2(\omega), \quad b(\xi_\omega) = \sigma_1(\omega)\xi_\omega + \sigma_2(\omega) \tag{25}$$

The integration scheme is therefore given by:

$$\begin{aligned} \xi_0^{(n)} &= \xi_0 \\ \xi_{\omega_j}^{(n)} &= \xi_{\omega_{j-1}}^{(n)} + (c_1(\omega_{j-1})\xi_{\omega_{j-1}}^{(n)} + c_2(\omega_{j-1}))\Delta\omega \\ &\quad + (\sigma_1(\omega_{j-1})\xi_{\omega_{j-1}}^{(n)} + \sigma_2(\omega_{j-1}))\Delta_j B \\ &\quad + \frac{1}{2}(\sigma_1(\omega_{j-1})\xi_{\omega_{j-1}}^{(n)} + \sigma_2(\omega_{j-1}))\sigma_1(\omega_{j-1})\{(\Delta_j B)^2 - \Delta\omega\} \end{aligned} \tag{26}$$

in which, the supper suffix (n) means that this solution is valid only for the case of the number of discrete points being n.

The concerned circular frequency range ω_D is segmented by equal interval of $\Delta\omega$. If the total number of discrete points is N, then $\Delta\omega$ is given by:

$$\Delta\omega = \frac{\omega_D}{N} \tag{27}$$

and the circular frequency at jth discrete point is given by

$$\omega_j = \Delta\omega \cdot j \tag{28}$$

Because B is the Wiener process, its stochastic differential $\Delta_j B = B(\omega_j) - B(\omega_{j-1})$ is a random variable expressed by a normal

Table 2. Results of Regression Analysis for Eq. (21) and Eq. (22) (ω_0 = 0 Hz)

i	M = 7.1		
	α_i	β_i	γ_i
1	-0.001	0.900	0.768
2	-0.002	0.900	1.045
3	6.600E-06	0.900	0.263
4	-0.014	0.900	-0.867

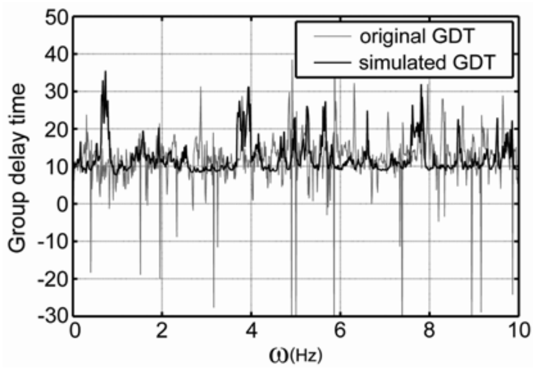

Fig. 3. Simulated Group Delay Time

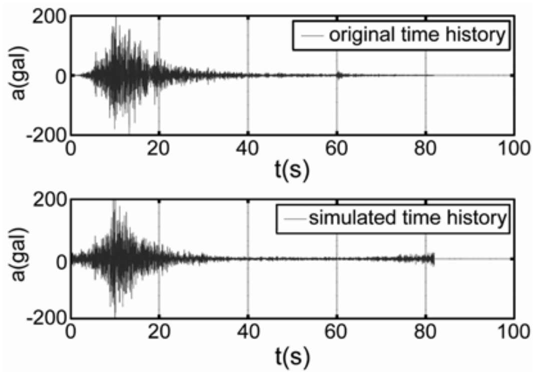

Fig. 4. Original and Simulated Time Histories

distribution with 0 mean and standard deviation $(\Delta\omega)^{1/2}$. The simplest way to generate $\Delta_j B$ is to multiply the value $(\Delta\omega)^{1/2}$ to a random number y_j ($j = 1,2,\ldots,$N) generated from the standard normal distribution with 0 mean and standard deviation of 1.0. So the value of $\Delta_j B$ is given by:

$$\Delta_j B = \sqrt{\Delta\omega} \cdot y_j \quad (29)$$

Then sample path of group delay time can be simulated based on the Milstein approximation. Fig. 3 shows one sample path of simulated group delay time of FKOH03-EW wave as well as the original one. A sample phase spectrum $\varphi(\omega)$ is obtained by integrating a sample group delay time with respect to frequency. Using the obtained phase spectrum and the original amplitude of FKOH03-EW wave, a simulated time history can be synthesized by:

$$a_s(t) = \mathrm{IFFT}(A_0(\omega)\mathrm{e}^{i\varphi(\omega)}) \quad (30)$$

Where, $a_s(t)$ is simulated time history, $A_o(\omega)$ is original amplitude, and $\varphi(\omega)$ is simulated phase spectrum. IFFT is inverse Fast Fourier Transform. It can be seen from Fig. 4 that the simulated time history is close to the original one.

4. Conclusions

In this paper a method to simulate group delay time of earthquake motion is developed by using stochastic differential equation. The stochastic characteristics of simulated GDT are validated by comparing them with those of observed earthquake motions. Coefficients of the stochastic differential equation are proposed as functions of earthquake magnitude, epicentral distance and circular frequency. The results of regression analysis show that influence of epicentral distance on coefficients increases as frequency increases. After the coefficients are identified, the Milstein approximation scheme is used to solve the stochastic differential equation of GDT. The efficiency of the developed method to simulate GDT of earthquake motion is demonstrated after the stochastic characteristics of simulated group delay times are checked. The efficiency of the simulation model is also demonstrated by comparing simulated time history with the original one. Stochastic characteristics of GDT checked in this paper show that the phase uncertainty should be modeled by the Wiener process.

In earthquake engineering analyzing, if epicentral distance between local site and potential seismic source is known, coefficients controlling stochastic differential equation could be identified by using local coefficients regression surfaces which might be established using the method given in this study in advance. Then GDT as well as phase spectrum could be simulated using the GDT model given. Usually amplitude spectrum could be simulated from some known amplitude spectrum models. With simulated amplitude and phase spectrum, time history of local site could be synthesized easily. The GDT model of earthquake motion not only gives a good physical interpretation for earthquake phase uncertainty, but also leads to a feasible and easy way to simulation of local site earthquake time history.

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